INDIAN SCHOOL AL WADI AL KABIR ISWK/P1/041/2

Pre – Board Examination 1 (2023 – 2024)

Class: XII Sub: MATHEMATICS (041) Max Marks: 80

Date: 30.11.2023 SET - 2 Time: 3 Hours

General Instructions:

1. This question paper is divided in to 5 sections- A, B, C, D and E
2. Section A comprises of 20 MCQ type questions of 1 mark each.
3. Section B comprises of 5 Very Short Answer Type Questions of 2 marks each.
4. Section C comprises of 6 Short Answer Type Questions of 3 marks each.
5. Section D comprises of 4 Long Answer Type Questions of 5 marks each.
6. Section E comprises of 3 source based / case based / passage-based questions (4 marks each) with sub parts.
7. Internal choice has been provided for certain questions

SECTION – A

(Each MCQ Carries 1 Mark)

1 If $f'(x) = x + \frac{1}{x}$, then *f(x)* is

a) x^2 + log |x| + c b) $\frac{x^2}{2}$ + log |x| + c c) $\frac{x}{2}$ - log |x| + c d) $\frac{x}{2}$ + log |x| + c

2 The value of '*k*' for which the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & if\ x \neq 0 \\ k, & if\ x = 0 \end{cases}$ is continuous at x = 0 is

a) 1 b) -1 c) 0 d) 2

3 If A = $\begin{bmatrix} -2 & 3 \\ k & 4 \end{bmatrix}$ is a singular matrix, then the value of 'k' is

a) $\frac{3}{8}$ b) $-\frac{3}{8}$ c) $\frac{8}{3}$ d) $-\frac{8}{3}$

4 If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then AA^T is

a) I b) 0 c) A d) - A

5 Corner points of the feasible region for an LPP are (0,2), (3,0), (6,0), (6,8) and (0,5). Let F = 4x + 6y be the objective function, then Maximum of F – Minimum of F = ______

a) 18 b) 42 c) 60 d) 78

6 The integrating factor for the given differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1 + x^2)}$ is

a) $\log(1 + x^2)$ b) 2x   c) $(1 + x^2)$ d) $\tan^{-1} x$

7 $\int e^{-x} . x$ dx equals

a) $xe^{-x} + e^{-x} + c$ c) $xe^{-x} - e^{-x} + c$

b) $- xe^{-x} - e^{-x} + c$ d) $- xe^{-x} + e^{-x} + c$

8 If $[x - 2 \quad 5+y]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = 0$, then x + y is

a) -4   b) -3   c) -2   d) -1

9 Let $\sin^{-1}(2x) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$. Then the value of *'x'* is

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{8}$ d) 8

10 If A is a square matrix of order 2 and |A| = - 3, then the value of |5A| is

a) -3   b) -75   c) -125   d) None of these

11 Shape of the feasible region formed by the constraints $x + y \leq 2$, $x + y \geq 5$, $x \geq 0$, $y \geq 0$ is

a) Triangular region   c) Unbounded solution

b) No feasible region   d) Quadrilateral

12 The order of differential equation $\frac{d^4y}{dx^4} + \sin\left(\frac{d^2y}{dx^2}\right) = 0$ is

a) 1   b) 2   c) 4   d) Not Defined

13 For the function $y = \frac{(x-7)}{(x-2)(x-3)}$, the value of $\left(\frac{dy}{dx}\right)_{\text{at } x = 7}$ is

a) - 20   b) $\frac{-1}{20}$ c) 20   d) $\frac{1}{20}$

14 If, $\vec{a} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} - 3\hat{k}$ then the value of $\vec{a} . \vec{b}$ is

a) -3   b) 3   c) 4   d) None of these

15 The direction cosines of the line joining (4, 3, -5) and (-2, 1, -8) are:

a) $\left(\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right)$ b) $\left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$ c) $\left(-\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$ d) None of these

16 Given two independent events A and B such that P(A) = 0.3, P(B) = 0.6 and $P(A' \cap B')$ is

a) 0.9 b) 0.10 c) 0.18 d) 0.28

17 If the x-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, -2) is 4. then its z-coordinate is

a) 0 b) - 1 c) - 9 d) None of these

18 If α, β, γ are the angle which a half ray makes with the positive directions of the axis then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

a) -1 b) 0 c) 1 d) 2

Directions: In the following 2 questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as.
(A) Both A and R are true and R is the correct explanation of A
(B) Both A and R are true but R is NOT the correct explanation of A
(C) A is true but R is false
(D) A is false and R is True

19 **Assertion (A):** If A = $\begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$, then A^{-1} does not exists

Reason (R): If A is a skew symmetric matrix of odd order, then A is singular

a) b) c) d)

20 **Assertion (A):** Direction cosines of a line are the sines of the angles made by the line with negative directions of coordinate axes.
Reason (R): the acute angle between the lines $x - 2 = 0$ and $\sqrt{3}x - y - 2 = 0$ is $30°$

a) b) c) d)

SECTION – B

(Each Question Carries 2 Marks)

21 Find the value of $sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$

- **OR** -

Find the value of $sin\left(cos^{-1}\frac{3}{5}\right) \cdot \sin\left(\frac{\pi}{2} - sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

22 A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, then find $\vec{r}$

- **OR** -

Using vectors, find the area of the triangle ABC with vertices A(1,2,3) , B(2,-1,4) and C(4,5,-1)

23 An edge of a variable cube is increasing at the rate of 3cm/sec. How is the volume of the cube increasing when the edge is 10cm long?

24 Find the value of λ if $(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}) = \vec{0}$

25 Find $\frac{dy}{dx}$ if x = a $\left(\cos t + \log \tan \frac{t}{2}\right)$ and y = a sin t

SECTION – C

(Each Question Carries 3 Marks)

26 Evaluate $\int_1^3 |x^3 - 2x|\, dx$

27 Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

- OR -

Evaluate $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}\, dx$

28 Solve the following Linear Programming Problem graphically: Maximise Z = 5x + 3y; subject to: $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

29 In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.

© If she reads English newspaper, find the probability that she reads Hindi newspaper.

- OR -

Suppose a girl throws a dice. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die?

30 Evaluate $\int \frac{x^2 + 1}{x^2 - 5x + 6}\, dx$

31 If y = 3 cos (log x) + 4 sin (log x), show that $x^2 y_2 + xy_1 + y = 0$

- OR -

If $y = x^x$, then show that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

SECTION – D

(Each Question Carries 5 Marks)

32 For real numbers x and y, we write xRy, implying that $x - y + \sqrt{2}$ is an irrational number. Then, check relation R is reflexive, symmetric, or transitive? Justify your answer.

\- OR -

Let $A = \{x \in Z: 0 \leq x \leq 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b|\ is\ divisible\ by\ 4\}$ is an equivalence relation. Find the set of all elements related to 1.

33 Find the shortest distance between the lines whose vector equations are
$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \boldsymbol{\lambda}(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$ and $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

\- OR -

If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, then find a vector $\vec{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$

34 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises of hard workers, the second group has honest and law abiding students and the third group contains obedient students. Double the number of students of the first group added to the number in the second group gives 13, while the combined strength of the first and second group is four times that of the third group. Using matrix method, find the number of students in each category.

35 Using the method of integration find the area bounded by the curve $x^2 = 4y$ and the line
$x = 4y - 2$

SECTION – E

(CASE STUDY – Each Question Carries 4 Marks)

36 Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by
$f(x) = 3x^4 + 4x^3 - 12x^2 + 12, -3 \leq x \leq 3$, where
f(x) is the temperature in °F at x days.

(i) Is the function differentiable in the interval (-3, 3)? Justify your answer. (1m)

(ii) Find the critical points. (1m)

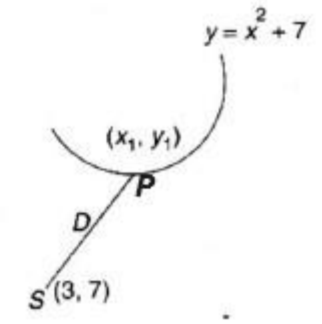
(iii) Find the intervals in which the function is strictly increasing (2m)

\- OR -

(iii) Find the absolute maximum / absolute minimum values of the function in the interval [-3,3]. (2m)

37 Read the following passage and answer the questions given below.

A helicopter of the enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7) want to shoot down the helicopter when it is nearest to him.



(i) If P (x_1, y_1) be the position of a helicopter on curve $y = x^2 + 7$, then find distance D from P to soldier place at (3, 7). (1m)

(ii) Find the critical point such that distance is minimum. (1m)

(iii) Verify by second derivative test that distance is minimum (2m)

\- OR -

(iii) Find the minimum distance between soldier and helicopter? (2m)

38 Senior students tend to stay up all night and therefore are not able to wake up on time in morning. Not only this but their dependence on tuitions further leads to absenteeism in school. Of the students in class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the class XII.

a) Find the probability of attaining A grade by the students of class XII (2m)

b) Find the probability that student is irregular given that he attains A grade. (2m)

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